

Kinematics

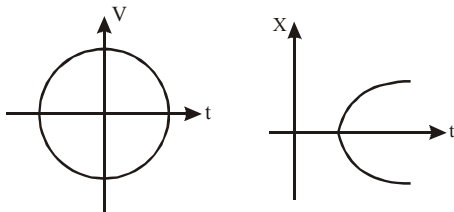
Conceptual Question :

1. What is meant by $\frac{d|\bar{v}|}{dt}$ and $\left|\frac{d\bar{v}}{dt}\right|$? Can these be equal? Can

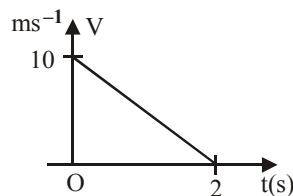
(i) $\frac{d|\bar{v}|}{dt} = 0$ while $\left|\frac{d\bar{v}}{dt}\right| \neq 0$

(ii) $\frac{d|\bar{v}|}{dt} \neq 0$ while $\left|\frac{d\bar{v}}{dt}\right| = 0$

2. Two balls of different masses m_1 and m_2 are dropped from two different heights h_1 and h_2 respectively. The ratio of time taken by the two balls to drop through these distances is _____.
3. Identify carefully and state with reasons, which of the following graphs cannot possibly represent one-dimensional motion of a particle.



4. For the velocity-time graph shown in the fig.
- (i) The initial velocity of the particle is _____.
- (ii) The acceleration of the particle is _____.
- (iii) If the particle starts from the origin, the displacement of the particle at $t = 2s$ is _____.



5. (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where t is in seconds and x is in meters), what is its velocity at $t = 1$ s?
 (b) Is it moving toward increasing or decreasing x just then?
 (a) What is its speed just then?
 (b) Is the speed larger or smaller at later times? (Try answering the next two questions without further calculation.)
 (c) Is there ever an instant when the velocity is zero?
 (d) Is there a time after $t = 3s$ when the particle is moving leftward on the x -axis?
6. A man standing on the edge of a cliff throws a stone straight up with initial speed u and then throws another stone straight down with same initial speed u and from the same position. Find the ratio of the speeds, the stones would have attained when they hit the ground at the base of the cliff.
7. A body dropped from a height h at time $t = 0$ reaches the ground at time t_0 . It would have reached a height $h/2$ at time $t =$ _____.
8. A body is thrown vertically up from the ground with a speed u and it reaches its maximum height h at time $t = t_0$. The height to which it would have risen at $t = t_0/2$ is _____.
9. Four persons K, L, M, N are initially at the four corners of a square of side d . Each person now moves with a uniform speed v in such a way that K always moves directly towards L, L directly towards M, M directly towards N and N directly towards K. The four persons will meet at a time _____.

True – False

State whether the statement is true or false.

1. When a particle is in motion, its acceleration may be in any direction.

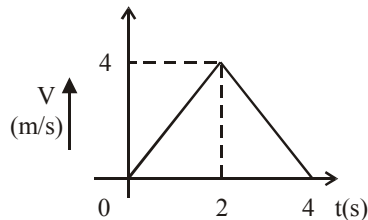
- A body can have zero velocity but non-zero acceleration.
- A body feels weightless in a projectile motion.
- The instantaneous magnitude of velocity is equal to the slope of the tangent drawn at the trajectory of the particle at that instant.

- A body moving with a uniform velocity in one frame A, then there is another frame B in which it is accelerating.
- A lorry and a car moving with the same kinetic energy are brought to rest by the application of brakes which provide equal retarding forces. Both come to rest in equal distances.

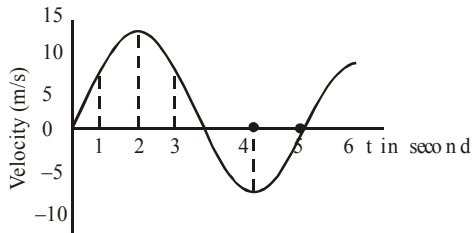
Part A

Motion In 1 – D

- The velocity-time graph of a particle moving along a straight line is shown in Fig.



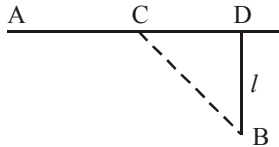
- If the particle starts its motion from $x = -4$ m, then draw the $a-t$ and $x-t$ graphs.
 - Find the displacement of the particle at $t = 3$ s.
- An object moves in a straight line as described by the velocity – time graph of fig. Sketch a graph that represents the acceleration of the object as a function of time.



- If the position of a particle is given by $x = 20t - 5t^3$, where x is in meters and t is in seconds, when if ever, is the particle's velocity zero?
 - When is its acceleration zero?
 - When is the acceleration negative?
 - When is the acceleration positive?
 - Graph $x(t)$, $v(t)$, and $a(t)$.

- Juggler tosses balls vertically a certain distance into the air. How much higher must they toss if they are to spend twice as much time in the air?
- Water drips from the nozzle of a shower onto the floor 200 cm below. The drops fall at regular intervals of time, the first drop striking the floor at the instant the fourth drop begins to fall. Find the locations of the second and third drops when the first strikes the floor.
- A parachutist bails out and freely falls 50 m. then the parachute opens, and thereafter he decelerates at 2.0 m/s^2 . He reaches the ground with a speed of 3.0 m/s .
 - How long is the parachutist in the air?
 - At what height did the fall begin?
- The position of a particle as it moves along the y -axis is given by $y = 2.0 \sin(\pi t/4)$, where t is in seconds and y is in centimeters.
 - What is the average velocity of the particle between $t = 0$ and $t = 2.0$ s?
 - What is the instantaneous velocity of the particle at $t = 0, 1.0$ and 2.0 s?
 - What is the average acceleration of the particle between $t = 0$ and $t = 2.0$ s?
 - What is the instantaneous acceleration of the particle at $t = 0, 1.0$ and 2.0 s?
 - Plot v versus t for $0 \leq t \leq 2.0$ s. and estimate the instantaneous acceleration at $t = 1.0$ s from the graph.
- A point moving with constant acceleration from A to B in the straight line AB has velocities u and v at A and B respectively. Find its velocity at C, the mid-point of AB. Also show that if the time from A to C is twice that from C to B, then $v = 7u$.

9. A car accelerates from rest at a constant α rate for sometime after which it decelerates at a constant rate β to come to rest. If the total time lapse is t seconds, evaluate
 (i) the maximum velocity reached and
 (ii) the total distance traveled.
10. Two mirrors, mounted vertically, are made to move towards each other with a speed v m/s each. A particle that can bounce back between the two mirrors starts from one mirror when the mirrors are d meters apart. On reaching the second mirror, it bounces back and so on. If the particle keeps on traveling at a constant speed of $3v$ m/s, how many trips can it make before the mirrors run into each other? What total distance does it cover?
11. If a body travels half of its total path in the last seconds of its fall from rest, find the time and height of its fall.
12. A particle is projected vertically upwards. Prove that it will be at $\frac{3}{4}$ of its greatest height at time, which are in the ratio 1 : 3.
13. From point A located on a highway one has to get car as soon as possible to point B located in the field at a distance l from the highway. It is known that the car moves in



The field n times slower than on the highway. At what distance from point D one must turn off the highway?

14. A point traversed half a circle of radius $R = 160$ cm during time interval $\tau = 10.0$ s. Calculate the following quantities averaged over that time:
 (a) The mean velocity $\langle \vec{v} \rangle$
 (b) The modulus of the mean velocity vector $|\langle \vec{v} \rangle|$;
 (c) The modulus of the mean vector of the total acceleration $|\langle \vec{w} \rangle|$ if the point moved with constant tangent acceleration.

Relative Velocity (1 – D)

15. Two particles A and B start their motion from the origin at the same instant. The acceleration of particle A is 1 ms^{-2} and that of particle B is 0.5 ms^{-2} .
 (i) Find the relative velocity between the two particles after 30 s.
 (ii) Find the relative distance between the two particles after 60 s.
16. A boat is traveling upstream at 14 km/h with respect to the water of the river. The water itself is flowing at 9 km/h with respect to the ground.
 (a) What is the velocity of the boat with respect to the ground?
 (b) A child on the boat walks from front to rear at 6 km/h with respect to the boat. What is the child's velocity with respect to the ground?
17. A person walks up a stationary escalator in t_1 seconds. If he remains stationary on the escalator, then it can take him up in t_2 seconds. If the length of the escalator is L , then
 (i) Determine the speed of man with respect to the escalator.
 (ii) Determine the speed of the escalator
 (iii) How much time would it take him to walk up the moving escalator?
18. Two cars, A and B, are traveling in the same direction with velocities v_A and v_B respectively. When car A is at distance d behind car B, the brakes on A are applied, causing a deceleration at the rate a . Show that to prevent a collision between A and B it is necessary that

$$(v_A - v_B)^2 < (2ad).$$
19. Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of $u = 20$ km/h in the direction A to B notices that a bus goes past him every $t_1 = 18$ minutes in the direction of his motion, and every $t_2 = 6$ minutes in the opposite direction. What is the period T of the bus service and with what speed do the busses ply on the road? Assume that velocity of bus is greater than that of the cyclist.

20. An elevator is descending with uniform acceleration. To measure the acceleration, a person in the elevator drops a coin at the moment the elevator starts. The coin is 6 ft above the floor of the elevator at the time it is dropped. The person observes that the coin strikes the floor in 1 second. Calculate from these data the acceleration of the elevator.
21. An elevator car whose floor-to-ceiling distance is equal to 2.7 m starts ascending with constant acceleration 1.2 m/s^2 ; 2.0 s after the start a bolt begins falling from the ceiling of the car. Find:
 (a) the bolt's free fall time;
 (b) the displacement and the distance covered by the bolt during the free fall in the reference frame fixed to the elevator shaft.
22. A police van moving on a highway with a speed of 30 km/h fires a bullet at thief's car speeding away in the same direction with a speed of 192 km/hr. if the muzzle speed of the bullet is 150 ms^{-1} , with what speed does the bullet hit the thief's car?

Relative Velocity (2 – D)

23. A helicopter is flying in a straight line over a level field at a constant speed of 6.2 m/s and at a constant altitude of 9.5 m. A package is ejected horizontally from the helicopter with an initial velocity of 12 m/s relative to the helicopter, and in a direction opposite the helicopter's motion.
 (a) Find the initial speed of the package relative to the ground.
 (b) What is the horizontal distance between the helicopter and the package at the instant the package strikes the ground?
 (c) What angle does the velocity vector of the package make with the ground at the instant before impact, as seen from the ground?
24. A train travels due south at 30 m/s (relative to ground) in a rain that is blown towards the south by the wind. The path of each raindrop makes an angle of 22° with the vertical, as measured by an observer stationary on the Earth. An observer on the train, however, sees the drops fall perfectly vertically. Determine the speed of the raindrops relative to the Earth.
25. A woman can row a boat 4.0 mi/h in still water.
 (a) If she is crossing a river where the current is 2.0 mi/h, in the direction must her boat be headed if she wants to reach a point directly opposite her starting point?
 (b) If the river is 4.0 mi wide, how long will it take her to cross the river?
 (c) Suppose that instead of crossing the river she rows 2.0 mi down the river and then back to her starting point. How long will she take?
 (d) How long will she take to row 2.0 mi up the river and then back to her starting point?
 (e) In what direction should she head the boat if she wants to cross in the shortest possible time, and what is that time?

Motion in 2 – D

26. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ be the angle of projection, prove that $\tan \theta = \tan \alpha + \tan \beta$.
27. Two positions A and B at the same height h above the ground. If the maximum height of the projectile is H , then determine the time t elapsed between the positions A and B in terms of H .
28. A point moves in the plane xy according to the law $x = kt$ and $y = kt(1 - \alpha t)$, where k and α are positive constants and t is time. Find
 (i) The equation of the points trajectory $y(x)$
 (ii) The velocity v and acceleration a of the point as function of time.
29. A particle is projected up an inclined plane of inclination β at an elevation α to the horizon. Show that
 (i) $\tan \alpha = \cot \beta + 2 \tan \beta$, if the particle strikes the plane at right angles;
 (ii) $\tan \alpha = 2 \tan \beta$ if the particle strikes the plane horizontally.
30. A stone is projected from the point of a ground in such a direction so as to hit a bird on the top of the telegraph post of height h and then attain the maximum height $2h$ above the ground. If at

he instant of projection, the bird was to fly horizontally.

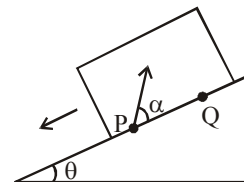
31. The velocity of a particle when it is at its greatest height is $\sqrt{(2/5)}$ of its velocity when it is at half its greatest height. Show that the angle of projection is 60° .
32. Two particles move in a uniform gravitational field with an acceleration g . At the initial moment the particles were located at one point

and moved with velocities $v_1 = 3.0$ m/s and $v_2 = 4.0$ m/s horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular.

Part – B

1. A particle is moving in a plane with velocity given by $\vec{u} = u_0 \hat{i} + (a \omega \cos \omega t) \hat{j}$, where \hat{i} and \hat{j} are unit vectors along x and y axes respectively. If particle is at the origin at $t = 0$.
 - (i) Calculate the trajectory of the particle.
 - (ii) Find the distance from the origin at time $3\pi/2\omega$. (Roorkee 1985)
2. A particle of mass 10^{-2} kg is moving along the positive x -axis under the influence of a force $F(x) = -K / 2x^2$, where $K = 10^{-2}$ Nm². At time $t = 0$, it is at $x = 1.0$ m and its velocity is $v = 0$.
 - (i) Find the velocity when it reaches $x = 0.50$ m
 - (ii) Find the time at which it reaches $x = 0.25$ m. (I.I.T. 1998)
3. A body falling freely from a given height H hits an inclined plane is its path at a height h . As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of (h/ H) the body will take maximum time to reach the ground? (I.I.T. 1986)
4. Two guns, situated of the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and other fire upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find (I.I.T. 1996)
 - (i) The time interval between the firings, and
 - (ii) The coordinates of the point P. Take origin of the coordinates system at the foot of the hill right below the muzzle and trajectories in X-Y plane.

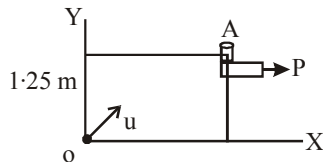
5. Particles P and Q of masses 20gm and 40 gm respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q make 45° and 135° angles respectively with the horizontal. Each particle has an initial speed of 49 m/sec. The separation AB is 245 m. Both particles travel in the same vertical plane and undergo a collision P retraces its path. Determine the position of Q when it hits the ground. How much times after the collision does the particle Q takes reach the ground. Take $g = 9.8$ m/sec². (I.I.T. 1982)
6. A gun kept on a straight horizontal road is used to hit a car, traveling along the same road away from the gun with a uniform speed of 72 km/hour. The car is at a distance of 500 meter from the gun, when the gun is fired at an angle of 45° with the horizontal. Find
 - (i) The distance of the car from the gun when the shell hits it.
 - (ii) The speed of projection of the shell from the gun. (I.I.T. 1974)
7. A large heavy box is sliding without friction down a smooth plane of inclination θ . from a point P on the bottom of a box; a particle is projected inside the box. The initial speed of the particle with respect to a box is u and the direction of projection makes an angle α with the bottom as shown in fig.



- (i) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).
- (ii) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

(I.I.T. 1998)

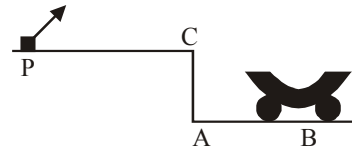
8. An object A is kept fixed at the point $x = 3\text{m}$ and $y = 1.25\text{m}$ on a plank P raised above the ground. At time $t = 0$ the plank starts moving along the $+x$ direction with an acceleration 1.5m/s^2 .



At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the x - y plane. Find u and the time after which the stone hits the object. Take $g = 10\text{m/s}^2$.

(I.I.T. 2000)

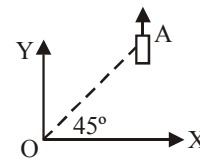
9. A car P is moving with a uniform speed of 53m/s towards a carriage of mass 9kg at rest kept on the rails at a point B as shown in fig. The height AC is 120m . Cannon balls of 1kg is fired from the car with an initial velocity 100m/s at an angle 30° with the horizontal. The first cannon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0 .



At t_0 , the second ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact?

(I.I.T. 2001)

10. On a frictionless horizontal surface, assumed to be the x - y plane, a small trolley A is moving along a straight line parallel to the y -axis (fig.) with a constant velocity of $(\sqrt{3}-1)\text{m/s}$. At a particular instant, when the line OA makes an angle of 45° with the x -axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x -axis and it hit the trolley.



- (i) The motion of the ball is observed from the frame of the trolley. Calculate the angle q made by the velocity vector of the ball with the x -axis in this frame.
- (ii) Find the speed of the ball with respect to the surface, if $\phi = 40/3$.

(I.I.T. 2002)

Part – C

1. The velocity of a particle moving in the positive direction of x -axis varies as $v = \alpha\sqrt{x}$ where α is positive constant. Assuming that at the moment $t = 0$, the particle was located at $x = 0$ find,
- (i) The time dependence of the velocity and the acceleration of the particle and

- (ii) The mean velocity of the particle averaged over the time that the particle takes to cover first s meters of the path.

2. A gun of muzzle speed v_0 is situated at height h above a horizontal plane. Prove that the angle at which it must be fired so as to achieve the greatest range on the plane given by

$$\theta = \frac{1}{2} \cos^{-1} \left(\frac{gh}{v_0^2 + gh} \right)$$

3. The radii of the front and rear wheels of a carriage are a and b respectively and c is the distance between their axles. A dust particle driven from the highest point of the rear wheel is obtained to alight on the highest point of the front wheel. Show that the velocity of the carriage is

$$\left(\frac{g(c+b-a)(c+a-b)}{4(b-a)} \right)^{1/2}$$

4. A perfectly elastic particle is projected with a velocity V in a vertical plane through the line of greatest slope of an inclined plane of elevation α . If after striking the plane, the particle rebounds vertically, show that it will return to the point of projection at the end of time equal to

$$\frac{\left(\frac{6V}{g\sqrt{1+8\sin^2\alpha}} \right)}{\frac{6V}{g\sqrt{1+8\sin^2\alpha}}}$$

5. Show that the shortest route to cross a river is at right angles to the river and is D when v (velocity of swimmer) $> u$ (velocity of river water) and in the reverse case it is Du/v where D is the width of the river.
6. Two particles are projected simultaneously in the same vertical plane, i and j being horizontal and vertical unit vectors in that plane. The first particle is projected from the origin with velocity $nV(\cos \alpha \hat{i} + \sin \alpha \hat{j})$ and the second is projected from a position $(h \hat{i} + k \hat{j})$ where $h > 0, k > 0$ with velocity $V(-\cos \beta \hat{i} + \sin \beta \hat{j})$. Show that the particle cannot collide unless $\sin \beta < n \sin \alpha$ and if they collide prove that $\sin(\beta + \gamma) = n \sin(\alpha - \gamma)$ where $\tan \gamma = k/h$.

7. Two ships A and B originally at a distance $a = 3$ km from each other depart at the same time from a straight coastline. Ship A moves along a straight line perpendicular to the shore while ship B constantly heads from ship A with the same speed v_0 . After a sufficiently long interval B will obviously follow a maintaining a certain distance. Find this distance.
8. A ball falls freely from the height h onto an inclined plane forming an angle α with the horizon. Find the ratio of the successive range of the ball along the plane. Consider the impacts between the ball and the plane to be absolutely elastic.
9. A boat moves relative to water with a velocity, which is $n = 2.0$ times less than the river flow velocity. At what angle to the stream direction must the boat move to minimize drifting?
10. A point moves rectilinearly with deceleration whose modulus depends on the velocity v of the particle as $w = a\sqrt{v}$, where a is a positive constant. At the initial moment the velocity of the point is equal to v_0 . What distance will it traverse before it stops? What time will it take to cover that distance?
11. A point moves in the plane xy according to the law $x = at, y = at(1 - \alpha t)$, where a and α are positive constants, and t is time. Find:
 (i) The equation of the point's trajectory $y(x)$; plot this function;
 (ii) The velocity v and the acceleration w of the point as functions of time;
 (iii) The moment t_0 at which the velocity vector forms an angle $\pi/4$ with the acceleration vector.
12. A small body is thrown at an angle to the horizontal with the initial velocity v_0 . Neglecting the air drag, find:
 (i) the displacement of the body as a function at time $r(t)$;
 (ii) the mean velocity vector $\langle \bar{v} \rangle$ averaged over the first t seconds and over the total time of motion.