

## CIRCULAR MOTION

### Conceptual Questions: -

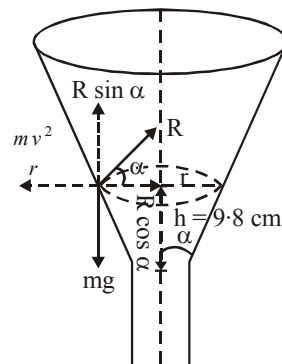
1. The earth is not a perfect sphere. Why does it bulges at the equator and flattens at the poles?
2. An automobile overturns when going too fast around a curve. Which wheels leave the ground first?

### PART – A

1. A motorcar traveling at 30 m/s on a circular road of radius 500 m. It is increasing its speed at the rate of  $2 \text{ m/s}^2$  what is the acceleration?
2. A thin circular wire of radius  $R$  rotates about its vertical diameter with an angular frequency  $\omega \leq \sqrt{(g/R)}$ .
3. A particle is attached by means of two equal strings to two points A and B in the same vertical line and describes a horizontal circle with a uniform angular speed. If the angular speed of the particle is  $2\sqrt{(2g/h)}$  with  $AB = h$ , show that the ratio of the tension of the string is 5 : 3.
4. Find the maximum speed at which a car can turn round a curve of 30 metre-radius on a level road if the coefficient of friction between the tyres and the road is 0.04. (Roorkee 1986)
5. A car goes on a horizontal circular road of radius  $R$ , the speed increasing at a constant rate  $\frac{dv}{dt} = a$ . The friction coefficient between the road and the tyre is  $\mu$ . Find the speed at which the car will skid.
6. A motorcycle has to move with a constant speed on an over bridge which is in the form of a circular arc of radius  $R$  and has a total length  $L$ . Suppose the motorcycle starts from the highest point. (a) What can its maximum velocity be for which the contact with the road is not broken at the highest point? (b) If the motorcycle goes at speed  $1/\sqrt{2}$  times the maximum found in part (a), where will it lose the contact with the road? (c) what maximum uniform speed can it maintain on the bridge if it does not lose contact anywhere on the bridge?

### PART – B

1. A particle describes a horizontal circle on the smooth inner surface of a conical funnel as shown in fig. If the height of the plane of the circle above the vertex is 9.8 cm, find the speed of the particle.



2. A string of length 1 m is fixed at one end and carries a mass of 100 gm at the other end. This string makes  $2/\pi$  revolution per second around a vertical axis passing through its second end. Calculate (a) the angle of inclination of a string with the vertical, (b) the tension in the string and (c) the linear velocity of the mass.

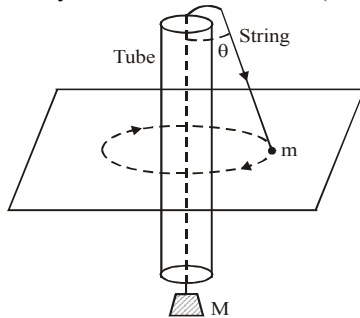
(I.I.T. 1976)

3. A 40kg mass, hanging at the end of a rope of length  $l$ , oscillates in a vertical plane with an angular amplitude of  $\theta_0$ . What is the tension in the rope, when it makes an angle  $\theta$  with the vertical? If the breaking strength of the rope is 80 kg f, What is the maximum angular amplitude with which the mass can oscillate without the rope breaking?

(I.I.T. 1978)

4. A large mass  $M$  and a small mass  $m$  hang at the two ends of a string that passes through a smooth tube as shown in fig. The mass  $m$  moves around in a circular path, which lies in the horizontal plane. The length of the string from the mass  $m$  to the top of the tube is  $l$  and  $\theta$  is the angle this length makes with the vertical. What should be the frequency of rotation of the mass  $m$  so that mass  $M$  remains stationary?

(I.I.T. 1978)



5. A hemispherical bowl of radius  $R = 0.1$  m is rotating about its own axis (which is vertical) with an angular velocity  $\omega$ . A particle of mass  $10^{-2}$  kg on the frictionless inner surface of the bowl is also rotating with same  $\omega$ . The particle is at a height  $h$  from the bottom of the bowl. (a) Obtain the relation between  $h$  and  $\omega$ . What is the minimum value of  $\omega$  needed in order to have a non-zero value of  $h$ ? (b) It is desired to measure  $g$  using this set up, by measuring  $h$  accurately. Assuming that  $r$  and  $\omega$  are known precisely, and that the least count in the measurement of  $h$  is  $10^{-4}$  m. What is the minimum possible error  $\Delta g$  in the measured value of  $g$ ?  $g = 9.8 \text{ m/s}^2$ .

(I.I.T. 1993)

6. A particle is suspended vertically from a point  $O$  by an inextensible mass less string of length  $L$ . A vertical line  $AB$  is at a distance  $L/8$  from

$O$  and is given a horizontal velocity  $u$ . At some point, its motion ceases to be circular and eventually the object passes through line  $AB$ . At the instant of crossing  $AB$ , its velocity is horizontal. Find  $u$ .

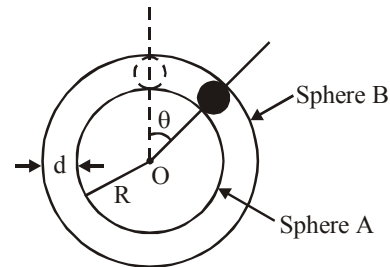
(I.I.T. 1999)

7. Two blocks of masses  $m_1 = 10\text{kg}$  and  $m_2 = 5\text{kg}$ , connected to each other by a mass less inextensible string of length  $0.3$  m are placed along a diameter of a turntable. The coefficient of friction between the table and  $m_1$  is  $0.5$  while there is no friction between  $m_2$  and table. The table is rotating with angular velocity  $10$  rad/sec about a vertical axis passing through its centre  $O$ . The masses are placed along the diameter of the table on either side of the centre  $O$  such that the mass  $m_1$  is at a distance of  $0.124$  m from  $O$ . The masses are observed to be at rest with respect to an observer on the turntable.

- Calculate the frictional force on  $m_1$ .
- What should be the minimum angular speed of the turntable so that the masses will slip from this position?
- How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass  $m_1$ ?

(I.I.T. 1997)

8. A spherical ball of mass  $m$  is kept at the highest point in the space between two fixed, concentric spheres  $A$  and  $B$  (see figure).



The smaller sphere  $A$  has a radius  $R$  and the space between the two spheres has a width  $d$ . The ball has a diameter very slightly less than  $d$ . All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by  $\theta$  (shown in the figure)

- Express the total normal reaction force exerted by the sphere on the ball as a function of angle  $\theta$ .
- Let  $N_A$  and  $N_B$  denote the magnitudes of the normal reaction forces on the ball exerted by the sphere  $A$  and  $B$ , respectively. Sketch the variations of  $N_A$  and  $N_B$  as functions of  $\cos \theta$  in the range  $0 \leq \theta \leq \pi$  by drawing two separate

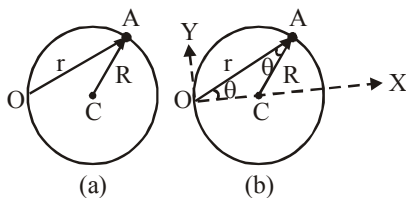
graphs in your answer book, taking  $\cos \theta$  the horizontal axis. (I.I.T. 2002)

9. A particle of mass 3 kg is moving under the action of a central force whose potential energy is given by  $U(r) = 10 r^3$  joule. For what energy and angular momentum will the

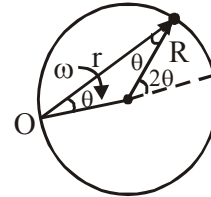
orbit be a circle of radius 10 m Calculate the time period of this motion? (Roorkee 1992)

### PART – C

1. A particle A moves along a circle of radius  $R = 50$  cm so that its radius vector  $r$  relative to point  $O$  (fig.) rotates with the constant angular velocity  $\omega = 0.40$  rad/sec. Find the modulus and direction of its total acceleration.



2. A solid body rotates about a stationary axis according to the law  $\phi = k t - b t^3$ , where  $k = 6.0$  rad/s and  $b = 2.0$  rad/s<sup>2</sup>. Find  
 (a) the mean values of the angular velocity and angular acceleration averaged over the time interval between  $t = 0$  and the complete stop;  
 (b) the angular acceleration at the moment when the body stops.
3. A small object slides without friction from the height  $H = 50$  cm shown in fig. And then loops the vertical loop of radius  $R = 20$  cm from which a symmetrical section of angle  $2\alpha$  has been removed. Find angle  $\alpha$  such that after losing contact at A and flying through the air, the object will reach at point B.
4. A point traversed half a circle of radius  $R = 160$  cm during time interval  $t = 10.0$ s (fig.) Calculate the following quantities averaged over that time:



- (a) mean velocity  $\bar{v}$ ;  
 (b) the modulus of mean velocity vector  $|\bar{v}|$   
 (c) the modulus of the mean vector of total acceleration  $|\bar{a}|$  if the point moved with constant tangential acceleration.
5. A table with smooth horizontal surface is turning at an angular speed  $\omega$  about its axis. A groove is made on the surface along a radius and a particle is gently placed inside the groove at a distance  $a$  from the centre. Find the speed of the particle with respect to the table, as its distance from the centre becomes  $L$ .
6. What is the radius of curvature of the parabola traced out by the projectile in the previous problem at a point where the particle velocity makes an angle  $\theta/2$  with the horizontal?
7. A heavy particle is suspended by a string of length  $l$ . The particle is given a horizontal velocity  $v_0$ . The string becomes slack at some angle and the particle proceeds on a parabola. Find the value of  $v_0$  if the particle passes through the point of suspension.
8. A smooth sphere of radius  $R$  is made to translate in a straight line with a constant acceleration  $a$ . A particle kept on the top of the sphere is released from there at zero velocity with respect to the sphere. Find the speed of the particle with respect to the sphere as a function of the angle  $\theta$  it slides.