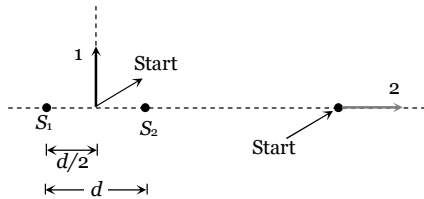


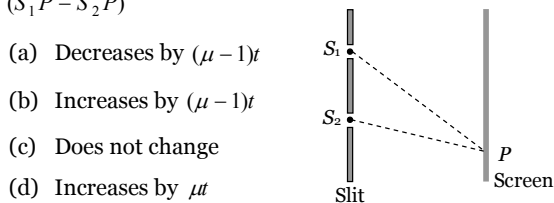
Wave Optics

Self Evaluation Test - 30

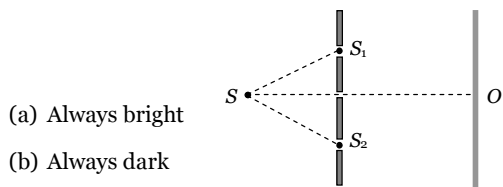
1. Following figure shows sources S_1 and S_2 that emits light of wavelength λ in all directions. The sources are exactly in phase and are separated by a distance equal to 1.5λ . If we start at the indicated start point and travel along path 1 and 2, the interference produce a maxima all along



- (a) Path 1
(b) Path 2
(c) Any path
(d) None of these
2. In a Young's double slit experimental arrangement shown here, if a mica sheet of thickness t and refractive index μ is placed in front of the slit S_1 , then the path difference ($S_1P - S_2P$)



- (a) Decreases by $(\mu - 1)t$
(b) Increases by $(\mu - 1)t$
(c) Does not change
(d) Increases by μt
3. In the set up shown in Fig the two slits, S_1 and S_2 are not equidistant from the slit S . The central fringe at O is then



- (a) Always bright
(b) Always dark
(c) Either dark or bright depending on the position of S
(d) Neither dark nor bright.
4. The intensity ratio of two coherent sources of light is p . They are interfering in some region and produce interference pattern. Then the fringe visibility is

- (a) $\frac{1+p}{2\sqrt{p}}$
(b) $\frac{2\sqrt{p}}{1+p}$
(c) $\frac{p}{1+p}$
(d) $\frac{2p}{1+p}$

5. Three waves of equal frequency having amplitudes $10\mu\text{m}$, $4\mu\text{m}$, $7\mu\text{m}$ arrive at a given point with successive phase difference of $\frac{\pi}{2}$, the amplitude of the resulting wave in μm is given by

- (a) 4
(b) 5
(c) 6
(d) 7

6. Four different independent waves are represented by

- (i) $y_1 = a_1 \sin \omega t$
(ii) $y_2 = a_2 \sin 2\omega t$

- (iii) $y_3 = a_3 \cos \omega t$
(iv) $y_4 = a_4 \sin\left(\omega t + \frac{\pi}{3}\right)$

With which two waves interference is possible

- (a) In (i) and (iii)
(b) In (i) and (iv)
(c) In (iii) and (iv)
(d) Insufficient data to predict.

7. A beam of light consisting of two wavelengths 650 nm and 520 nm is used to illuminate the slit of a Young's double slit experiment. Then the order of the bright fringe of the longer wavelength that coincide with a bright fringe of the shorter wavelength at the least distance from the central maximum is

- (a) 1
(b) 2
(c) 3
(d) 4

8. Two identical radiators have a separation of $d = \lambda/4$ where λ is the wavelength of the waves emitted by either source. The initial phase difference between the sources is $\pi/4$. Then the intensity on the screen at a distant point situated at an angle $\theta = 30^\circ$ from the radiators is (here I_o is intensity at that point due to one radiator alone)

- (a) I_o
(b) $2I_o$
(c) $3I_o$
(d) $4I_o$

9. In Young's double slit experiment, the 8th maximum with wavelength λ_1 is at a distance d_1 from the central maximum and the 6th maximum with a wavelength λ_2 is at a distance d_2 . Then (d_1 / d_2) is equal to

- (a) $\frac{4}{3} \left(\frac{\lambda_2}{\lambda_1} \right)$
(b) $\frac{4}{3} \left(\frac{\lambda_1}{\lambda_2} \right)$

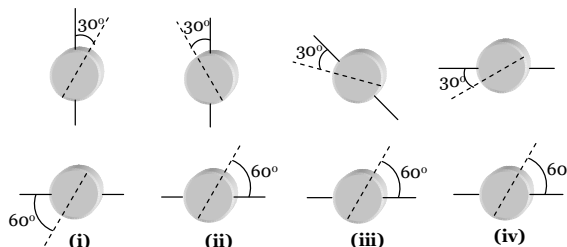
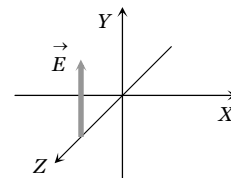
- (c) $\frac{3}{4} \left(\frac{\lambda_2}{\lambda_1} \right)$
(d) $\frac{3}{4} \left(\frac{\lambda_1}{\lambda_2} \right)$

10. Light of wavelength 500 nm is used to form interference pattern in Young's double slit experiment. A uniform glass plate of refractive index 1.5 and thickness 0.1 mm is introduced in the path of one of the interfering beams. The number of fringes which will shift the cross wire due to this is

- (a) 100
(b) 200

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- (c) 300 (d) 400
11. The two coherent sources of equal intensity produce maximum intensity of 100 units at a point. If the intensity of one of the sources is reduced by 36% by reducing its width then the intensity of light at the same point will be
 (a) 90 (b) 89
 (c) 67 (d) 81
12. The path difference between two interfering waves of equal intensities at a point on the screen is $\frac{\lambda}{4}$. The ratio of intensity at this point and that at the central fringe will be
 (a) 1 : 1 (b) 1 : 2
 (c) 2 : 1 (d) 1 : 4
13. In a Young's double slit experiment, I_o is the intensity at the central maximum and β is the fringe width. The intensity at a point P distant x from the centre will be
 (a) $I_o \cos \frac{\pi x}{\beta}$ (b) $4I_o \cos^2 \frac{\pi x}{\beta}$
 (c) $I_o \cos^2 \frac{\pi x}{\beta}$ (d) $\frac{I_o}{4} \cos^2 \frac{\pi x}{\beta}$
14. In a Fresnel's diffraction arrangement, the screen is at a distance of 2 meter from a circular aperture. It is found that for light of wavelengths λ_1 and λ_2 , the radius of 4th zone for λ_1 coincides with the radius of 5th zone for λ_2 . Then the ratio $\lambda_1 : \lambda_2$ is
 (a) $\sqrt{4/5}$ (b) $\sqrt{5/4}$
 (c) 5/4 (d) 4/5
15. If n represents the order of a half period zone, the area of this zone is approximately proportional to n^m where m is equal to
 (a) Zero (b) Half
 (c) One (d) Two
16. A screen is placed 50 cm from a single slit, which is illuminated with 6000 Å light. If distance between the first and third minima in the diffraction pattern is 3 mm, the width of the slit is
 (a) 0.1 mm (b) 0.2 mm
 (c) 0.3 mm (d) 0.4 mm
17. In Young's double slit experiment, the fringes are displaced by a distance x when a glass plate of one refractive index 1.5 is introduced in the path of one of the beams. When this plate is replaced by another plate of the same thickness, the shift of fringes is $(3/2)x$. The refractive index of the second plate is
 (a) 1.75 (b) 1.50
 (c) 1.25 (d) 1.00
18. Two waves of equal amplitude and frequency interfere each other. The ratio of intensity when the two waves arrive in phase to that when they arrive 90° out of phase is
 (a) 1 : 1 (b) $\sqrt{2} : 1$
 (c) 2 : 1 (d) 4 : 1
19. In Young's double slit experiment, we get 60 fringes in the field of view of monochromatic light of wavelength 4000 Å. If we use monochromatic light of wavelength 6000 Å, then the number of fringes obtained in the same field of view is
 (a) 60 (b) 90
 (c) 40 (d) 1.5
20. A parallel plate capacitor with plate area A and separation between the plates d , is charged by a constant current i , consider a plane surface of area $A/2$ parallel to the plates and drawn symmetrically between the plates, the displacement current through this area, will be.
 (a) i (b) $\frac{i}{2}$
 (c) $\frac{i}{4}$ (d) None of these
21. The figure here gives the electric field of an EM wave at a certain point and a certain instant. The wave is transporting energy in the negative z direction. What is the direction of the magnetic field of the wave at that point and instant
 (a) Towards + X direction
 (b) Towards - X direction
 (c) Towards + Z direction
 (d) Towards - Z direction



- (a) (i) > (ii) > (iii) > (iv) (b) (i) > (iv) > (ii) > (iii)
 (c) (i) > (iii) > (ii) > (iv) (d) (iv) > (iii) > (ii) > (i)

23. An astronaut floating freely in space decides to use his flash light as a rocket. He shines a 10 watt light beam in a fixed direction so that he acquires momentum in the opposite direction. If his mass is 80 kg, how long must he need to reach a velocity of 1 ms⁻¹
 (a) 9 sec (b) 2.4 × 10³ sec

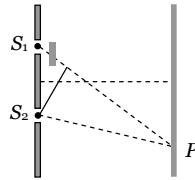
(c) $2.4 \times 10^6 \text{ sec}$ (d) $2.4 \times 10^9 \text{ sec}$

AS Answers and Solutions

(SET -30)

1. (a) At any point along the path 1, path difference between the waves is 0.
Hence maxima is obtained all along the path 1.
At any point along the path 2, path difference is 1.5λ which is odd multiple of $\frac{\lambda}{2}$, so minima is obtained all along the path 2.

2. (b) Path difference at P $\Delta = (S_1P + (\mu - 1)t) - S_2P$
 $= (S_1P - S_2P) + (\mu - 1)t$



3. (c) If path difference $\Delta = (SS_1 + S_1O) - (SS_2 + S_2O) = n\lambda$ $n = 0, 1, 2, 3, \dots$ the central fringe at O is a bright fringe and if the path difference $\Delta = \left(n - \frac{1}{2}\right)\lambda$, $n = 1, 2, 3, \dots$ the central bright fringe will be a dark fringe.

4. (b) Visibility $V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$
 $= \frac{2\sqrt{I_1 / I_2}}{\left(\frac{I_1}{I_2} + 1\right)} = \frac{2\sqrt{P}}{P + 1}$

5. (b) The amplitudes of the waves are

$$a_1 = 10 \mu\text{m}, a_2 = 4 \mu\text{m} \text{ and } a_3 = 7 \mu\text{m}$$

and the phase difference between 1st and 2nd wave is $\frac{\pi}{2}$ and that between 2nd and 3rd wave is $\frac{\pi}{2}$.

Therefore, phase difference between 1st and 3rd is π . Combining 1st with 3rd, their resultant amplitude is given by

$$A_1^2 = a_1^2 + a_3^2 + 2a_1a_3 \cos \phi$$

$$\text{or } A_1 = \sqrt{10^2 + 7^2 + 2 \times 10 \times 7 \cos \pi} = \sqrt{100 + 49 - 140}$$

$$= \sqrt{9} = 3 \mu\text{m} \text{ in the direction of first.}$$

Now combining this with 2nd wave we have, the resultant amplitude

$$A^2 = A_1^2 + a_2^2 + 2A_1a_2 \cos \frac{\pi}{2}$$

$$\text{or } A = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \cos 90^\circ} = \sqrt{9 + 16} = 5 \mu\text{m}$$

6. (d) Since the sources are independent, interference will not occur unless they are coherent (such as laser beams etc). So, insufficient data to predict.

7. (d) $n\beta_1 = (n+1)\beta_2$

$$\Rightarrow \frac{n \times 650 \times 10^{-19} D}{d} = \frac{(n+1) \times 520 \times 10^{-19} \times D}{d}$$

$$\Rightarrow n = 4$$

8. (b) The intensity at a point on screen is given by

$$I = 4I_0 \cos^2(\phi / 2)$$

where ϕ is the phase difference. In this problem ϕ arises (i) due to initial phase difference of $\pi/4$ and (ii) due to path difference for the observation point situated at $\theta = 30^\circ$. Thus

$$\phi = \frac{\pi}{4} + \frac{2\pi}{\lambda}(d \sin \theta) = \frac{\pi}{4} + \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} (\sin 30^\circ) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

$$\text{Thus } \frac{\phi}{2} = \frac{\pi}{4} \text{ and } I = 4I_0 \cos^2(\pi/4) = 2I_0$$

9. (b) Position of n^{th} maxima from central maxima is given by $x_n = \frac{n\lambda D}{d}$

$$\Rightarrow x_n \propto n\lambda \Rightarrow \frac{d_1}{d_2} = \frac{n_1 \lambda_1}{n_2 \lambda_2} = \frac{8 \lambda_1}{6 \lambda_2} = \frac{4}{3} \left(\frac{\lambda_1}{\lambda_2} \right)$$

10. (a) The number of fringes shifting is decided by the extra path difference produced by introducing the glass plate. The extra path difference is $(\mu - 1)t = n\lambda$ or $(1.5 - 1) \times 0.1 \times 10^{-3} = n \times 500 \times 10^{-9}$
 $\Rightarrow n = 100$

11. (d) Intensity of each source $= I_0 = \frac{100}{4} = 25 \text{ unit}$

If the intensity of one of the source is reduced by 36%

$$\text{then } I_1 = 25 \text{ unit and } I_2 = 25 - 25 \times \frac{36}{100} = 16 \text{ (unit)}$$

Hence resultant intensity at the same point will be

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} = 25 + 16 + 2\sqrt{25 \times 16} = 81 \text{ unit}$$

12. (b) By using $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right) = 4I_0 \cos^2\left(\frac{\pi\Delta}{\lambda}\right)$

$$\left\{ \because \phi = \frac{2\pi}{\lambda} \Delta \right\}$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{\cos^2\left(\frac{\pi\Delta_1}{\lambda}\right)}{\cos^2\left(\frac{\pi\Delta_2}{\lambda}\right)} = \frac{\cos^2\left(\frac{\pi \cdot \frac{\lambda}{4}}{\lambda}\right)}{\cos^2(0)} = \frac{1}{2}$$

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13. (c) Path difference at point $P = \frac{xd}{D}$
 Phase difference at point $P = \frac{2\pi}{\lambda} \frac{xd}{D} = \frac{2\pi x}{\beta}$
 $I_0 = 4I_1$, intensity at point P
 $I = I_1 + I_1 + 2I_1 \cos \frac{2\pi x}{\beta} = 2I_1 \left[1 + \cos \frac{2\pi x}{\beta} \right]$
 $= I_0 \cos^2 \frac{\pi x}{\beta}$
14. (c) It is given that $r_4 = \sqrt{4b\lambda_1}$ and $r_5 = \sqrt{5b\lambda_2}$
 are equal. Therefore $\sqrt{4b\lambda_1} = \sqrt{5b\lambda_2}$
 or $4b\lambda_1 = 5b\lambda_2$
 or $\frac{\lambda_1}{\lambda_2} = \frac{5}{4}$.
15. (a) Area of half period zone is independent of order of zone. Therefore, m is equal to zero in n^m .
16. (b) Position of n^{th} minima $y_n = \frac{n\lambda D}{d}$
 $\Rightarrow (y_3 - y_1) = \frac{\lambda D}{d} (3 - 1) = \frac{2\lambda D}{d}$
 $\Rightarrow 3 \times 10^{-3} = \frac{2 \times 6000 \times 10^{-10} \times 0.5}{d}$
 $\Rightarrow d = 0.2 \times 10^{-3} \text{ m} = 0.2 \text{ mm}$
17. (b) Fringe shift is given by $x = \frac{(\mu - 1)t\beta}{\lambda}$
 For first plate, $x = \frac{(\mu_1 - 1)t\beta}{\lambda}$
 For second plate $\frac{3}{2}x = \frac{(\mu_2 - 1)t\beta}{\lambda}$
 $\Rightarrow \left(\frac{\mu_2 - 1}{\mu_1 - 1} \right) = \frac{3}{2} \Rightarrow \left(\frac{\mu_2 - 1}{1.5 - 1} \right) = \frac{3}{2}$
 $\Rightarrow \mu_2 = 1.75$

18. (c) Resultant intensity $I = 4I_0 \cos^2(\phi/2)$
 $\Rightarrow \frac{I_1}{I_2} = \frac{\cos^2(\phi_1/2)}{\cos^2(\phi_2/2)} = \frac{\cos^2 0}{\cos^2(90/2)} = \frac{2}{1}$
19. (c) $n_1\lambda_1 = n_2\lambda_2 \Rightarrow 60 \times 4000 = n_2 \times 6000 \Rightarrow n_2 = 40$
20. (b) Suppose the charge on the capacitor at time t is Q , the electric field between the plates of the capacitor is $E = \frac{Q}{\epsilon_0 A}$. The flux through the area considered is $\phi_E = \frac{Q}{\epsilon_0 A} \cdot \frac{A}{2} = \frac{Q}{2\epsilon_0}$
 \therefore The displacement current
 $i_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \left(\frac{1}{2\epsilon_0} \right) \frac{dQ}{dt} = \frac{i}{2}$.
21. (a) The direction of EM wave is given by the direction of $\vec{E} \times \vec{B}$.
22. (b) Final intensity of light is given by Brewster's law $I = I_0 \cos^2 \theta$, where $\theta =$ Angle between transmission axes of polariser and analyser.
- (i) (ii) (iii) (iv)
-
- Hence decreasing order of intensity is (i) > (iv) > (ii) > (iii)
23. (d) Let it take t sec for astronaut to acquire a velocity of 1 ms^{-1} . Then energy of photons = $10 t$
 Momentum = $\frac{10t}{C} = 80 \times 1$
 $t = \frac{80 \times 1 \times 3 \times 10^8}{10} = 2.4 \times 10^9 \text{ sec}$
