Vectors

SET Self Evaluation Test - 0

1. $0.4\hat{i} + 0.8\hat{j} + c\hat{k}$ represents a unit vector when c is

   (a) – 0.2 (b) $\sqrt{0.2}$

   (c) $\sqrt{0.8}$ (d) 0

2. The angles which a vector $\hat{i} + \hat{j} + \sqrt{2}\,\hat{k}$ makes with X, Y and Z axes respectively are

   (a) 60°, 60°, 60° (b) 45°, 45°, 45°

   (c) 60°, 60°, 45° (d) 45°, 45°, 60°

3. The value of a unit vector in the direction of vector $A = 5\hat{i} - 12\hat{j}$, is

   (a) $\hat{i}$ (b) $\hat{j}$

   (c) $(\hat{i} + \hat{j})/13$ (d) $(5\hat{i} - 12\hat{j})/13$

4. Which of the following is independent of the choice of co-ordinate system

   (a) $\vec{P} + \vec{Q} + \vec{R}$ (b) $(P_x + Q_x + R_x)\hat{i}$

   (c) $P_x\hat{i} + Q_y\hat{j} + R_z\hat{k}$ (d) None of these

5. A car travels 6 *km* towards north at an angle of 45° to the east and then travels distance of 4 *km* towards north at an angle of 135° to the east. How far is the point from the starting point. What angle does the straight line joining its initial and final position makes with the east

   (a) $\sqrt{50}\ km$ and $\tan^{-1}(5)$

   (b) 10 *km* and $\tan^{-1}(\sqrt{5})$

   (c) $\sqrt{52}\ km$ and $\tan^{-1}(5)$

   (d) $\sqrt{52}\ km$ and $\tan^{-1}(\sqrt{5})$

6. Given that $\vec{A} + \vec{B} + \vec{C} = 0$ out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then the angles between vectors are given by

   (a) 30°, 60°, 90° (b) 45°, 45°, 90°

   (c) 45°, 60°, 90° (d) 90°, 135°, 135°

7. Two forces $F_1 = 1\,N$ and $F_2 = 2\,N$ act along the lines $x = 0$ and $y = 0$ respectively. Then the resultant of forces would be

   (a) $\hat{i} + 2\hat{j}$ (b) $\hat{i} + \hat{j}$

   (c) $3\hat{i} + 2\hat{j}$ (d) $2\hat{i} + \hat{j}$

8. At what angle must the two forces $(x + y)$ and $(x - y)$ act so that the resultant may be $\sqrt{(x^2 + y^2)}$

   (a) $\cos^{-1}\left(-\dfrac{x^2 + y^2}{2(x^2 - y^2)}\right)$ (b) $\cos^{-1}\left(-\dfrac{2(x^2 - y^2)}{x^2 + y^2}\right)$

   (c) $\cos^{-1}\left(-\dfrac{x^2 + y^2}{x^2 - y^2}\right)$ (d) $\cos^{-1}\left(-\dfrac{x^2 - y^2}{x^2 + y^2}\right)$

9. Following forces start acting on a particle at rest at the origin of the co-ordinate system simultaneously

   $\vec{F}_1 = -4\hat{i} - 5\hat{j} + 5\hat{k}$, $\vec{F}_2 = 5\hat{i} + 8\hat{j} + 6\hat{k}$, $\vec{F}_3 = -3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{F}_4 = 2\hat{i} - 3\hat{j} - 2\hat{k}$ then the particle will move

   (a) In $x - y$ plane (b) In $y - z$ plane

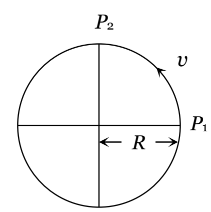
   (c) In $x - z$ plane (d) Along x -axis

10. The resultant of $\vec{A} + \vec{B}$ is $\vec{R}_1$. On reversing the vector $\vec{B}$, the resultant becomes $\vec{R}_2$. What is the value of $R_1^2 + R_2^2$

    (a) $A^2 + B^2$ (b) $A^2 - B^2$

    (c) $2(A^2 + B^2)$ (d) $2(A^2 - B^2)$

11. Figure below shows a body of mass M moving with the uniform speed on a circular path of radius, R. What is the change in acceleration in going from P_1 to P_2

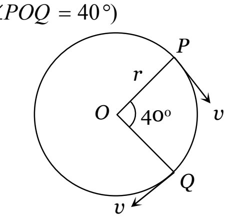
    (a) Zero

    (b) $v^2/2R$

    (c) $2v^2/R$

    (d) $\dfrac{v^2}{R} \times \sqrt{2}$

12. A particle is moving on a circular path of radius r with uniform velocity v. The change in velocity when the particle moves from P to Q is $(\angle POQ = 40°)$

    (a) $2v\cos 40°$

    (b) $2v\sin 40°$

    (c) $2v\sin 20°$

    (d) $2v\cos 20°$

13. $\vec{A} = 2\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} - 4\hat{k}$ are two vectors. The angle between them will be

    (a) 0° (b) 45°

    (c) 60° (d) 90°

14. If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$ then projection of $\vec{A}$ on $\vec{B}$ will be

(a) $\frac{3}{\sqrt{13}}$ (b) $\frac{3}{\sqrt{26}}$

(c) $\sqrt{\frac{3}{26}}$ (d) $\sqrt{\frac{3}{13}}$

15. In above example a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$ will be

(a) $+\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ (b) $-\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

(c) Both (a) and (b) (d) None of these

16. Two constant forces $F_1 = 2\hat{i} - 3\hat{j} + 3\hat{k}$ (*N*) and $F_2 = \hat{i} + \hat{j} - 2\hat{k}$ (*N*) act on a body and displace it from the position $r_1 = \hat{i} + 2\hat{j} - 2\hat{k}$ (m) to the position $r_2 = 7\hat{i} + 10\hat{j} + 5\hat{k}$ (m). What is the work done

(a) 9 *J* (b) 41 *J*

(c) – 3 *J* (d) None of these

17. For any two vectors $\vec{A}$ and $\vec{B}$, if $\vec{A}.\vec{B} = |\vec{A} \times \vec{B}|$, the magnitude of $\vec{C} = \vec{A} + \vec{B}$ is equal to

(a) $\sqrt{A^2 + B^2}$ (b) $A + B$

(c) $\sqrt{A^2 + B^2 + \frac{AB}{\sqrt{2}}}$ (d) $\sqrt{A^2 + B^2 + \sqrt{2} \times AB}$

18. Which of the following is the unit vector perpendicular to $\vec{A}$ and $\vec{B}$

(a) $\frac{\hat{A} \times \hat{B}}{AB\sin\theta}$ (b) $\frac{\hat{A} \times \hat{B}}{AB\cos\theta}$

(c) $\frac{\vec{A} \times \vec{B}}{AB\sin\theta}$ (d) $\frac{\vec{A} \times \vec{B}}{AB\cos\theta}$

19. Two vectors $P = 2\hat{i} + b\hat{j} + 2\hat{k}$ and $Q = \hat{i} + \hat{j} + \hat{k}$ will be parallel if

(a) $b = 0$ (b) $b = 1$

(c) $b = 2$ (d) $b = -4$

20. Which of the following is not true ? If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 6\hat{i} + 8\hat{j}$ where A and B are the magnitudes of $\vec{A}$ and $\vec{B}$

(a) $\vec{A} \times \vec{B} = 0$ (b) $\frac{A}{B} = \frac{1}{2}$

(c) $\vec{A}.\vec{B} = 48$ (d) A = 5

21. The area of the triangle formed by $2\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ is

(a) 3 sq.unit

(b) $2\sqrt{3}$ sq. unit

(c) $2\sqrt{14}$ sq. unit

(d) $\frac{\sqrt{14}}{2}$ sq. unit

22. Two trains along the same straight rails moving with constant speed 60 *km/hr* and 30 *km/hr* respectively towards each other. If at time $t = 0$, the distance between them is 90 *km,* the time when they collide is

(a) 1 *hr* (b) 2 *hr*

(c) 3 *hr* (d) 4 *hr*

23. A steam boat goes across a lake and comes back (a) On a quite day when the water is still and (b) On a rough day when there is uniform air current so as to help the journey onward and to impede the journey back. If the speed of the launch on both days was same, in which case it will complete the journey in lesser time

(a) Case (a)

(b) Case (b)

(c) Same in both

(d) Nothing can be predicted

24. To a person, going eastward in a car with a velocity of 25 *km/hr*, a train appears to move towards north with a velocity of $25\sqrt{3}$ *km/hr*. The actual velocity of the train will be

(a) 25 *km/hr* (b) 50 *km/hr*

(c) 5 *km/hr* (d) $5\sqrt{3}$ *km/hr*

25. A swimmer can swim in still water with speed υ and the river is flowing with velocity $v/2$. To cross the river in shortest distance, he should swim making angle θ with the upstream. What is the ratio of the time taken to swim across the shortest time to that is swimming across over shortest distance

(a) $\cos\theta$ (b) $\sin\theta$

(c) $\tan\theta$ (d) $\cot\theta$

26. A bus is moving with a velocity 10 m/s on a straight road. A scooterist wishes to overtake the bus in 100 *s*. If the bus is at a distance of 1 *km* from the scooterist, with what velocity should the scooterist chase the bus

(a) 50 *m/s* (b) 40 *m/s*

(c) 30 *m/s* (d) 20 *m/s*

Answers and Solutions

(SET -0)

1. (b) $\sqrt{(0.4)^2+(0.8)^2+c^2}=1$

$\Rightarrow 0.16+0.64+c^2=1 \Rightarrow c=\sqrt{0.2}$

2. (c) $\vec{R}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}$

Comparing the given vector with $R=R_x\hat{i}+R_y\hat{j}+R_z\hat{k}$

$R_x=1, R_y=1, R_z=\sqrt{2}$ and $|\vec{R}|=\sqrt{R_x^2+R_y^2+R_z^2}=2$

$\cos\alpha=\frac{R_x}{R}=\frac{1}{2}\Rightarrow\alpha=60°$

$\cos\beta=\frac{R_y}{R}=\frac{1}{2}\Rightarrow\beta=60°$

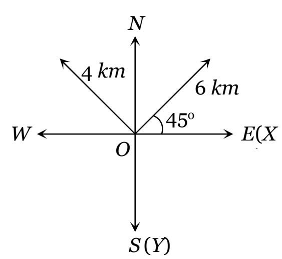
$\cos\gamma=\frac{R_z}{R}=\frac{1}{\sqrt{2}}\Rightarrow\gamma=45°$

3. (d) $\vec{A}=5\hat{i}+12\hat{j}, |\vec{A}|=\sqrt{5^2+(-12)^2}=\sqrt{25+144}=13$

Unit vector $\hat{A}=\frac{\vec{A}}{|\vec{A}|}=\frac{5\hat{i}-12\hat{j}}{13}$

4. (a)

5. (c)



Net movement along x-direction $S_x=(6-4)\cos 45°$

$\hat{i}$

$=2\times\frac{1}{\sqrt{2}}=\sqrt{2}\,km$

Net movement along y-direction $S_y=(6+4)\sin 45°\,\hat{j}$

$=10\times\frac{1}{\sqrt{2}}=5\sqrt{2}\,km$

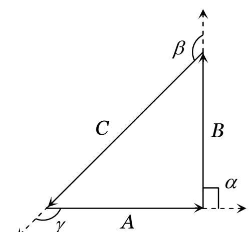
Net movement from starting point

$|\vec{S}|=\sqrt{S_x{}^2+S_y{}^2}=\sqrt{(\sqrt{2})^2+(5\sqrt{2})^2}=\sqrt{52}\,km$

Angle which makes with the east direction

$\tan\theta=\frac{\text{Y}-\text{component}}{\text{X}-\text{component}}=\frac{5\sqrt{2}}{\sqrt{2}}\;\therefore\;\theta=\tan^{-1}(5)$

6. (d)



From polygon law, three vectors having summation zero should form a closed polygon. (Triangle) since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. *i.e.* the triangle should be right angled triangle

Angle between A and B, $\alpha=90°$

Angle between B and C, $\beta=135°$

Angle between A and C, $\gamma=135°$

7. (d) $x=0$ means y-axis $\Rightarrow \vec{F}_1=\hat{j}$

$y=0$ means x-axis $\Rightarrow \vec{F}_2=2\hat{i}$

so resultant $\vec{F}=\vec{F}_1+\vec{F}_2=2\hat{i}+\hat{j}$

8. (a) $R^2=A^2+B^2+2AB\cos\theta$

Substituting, $A=(x+y)$, $B=(x-y)$ and $R=\sqrt{(x^2+y^2)}$

we get $\theta=\cos^{-1}\left(-\frac{(x^2+y^2)}{2(x^2-y^2)}\right)$

9. (b) $F_1+\vec{F}_2+\vec{F}_3+\vec{F}_4$

$=(-4\hat{i}+5\hat{i}-3\hat{i}+2\hat{i})+(-5\hat{j}+8\hat{j}+4\hat{j}-3\hat{j})$

$+(5\hat{k}+6\hat{k}-7\hat{k}-2\hat{k})=4\hat{j}+2\hat{k}$

$\therefore$ the particle will move in $y-z$ plane.

10. (c) $\vec{R}_1=\vec{A}+\vec{B}$, $\vec{R}_2=\vec{A}-\vec{B}$

$R_1^2+R_2^2=\left(\sqrt{A^2+B^2}\right)^2+\left(\sqrt{A^2+B^2}\right)^2=2\left(A^2+B^2\right)$

11. (d) $\Delta a=2a\sin\left(\frac{\theta}{2}\right)=2a\times\sin 45°=\sqrt{2}a=\sqrt{2}\frac{v^2}{R}$

12. (b) $\Delta v=2v\sin\left(\frac{\theta}{2}\right)=2v\sin 20°$

13. (c) $\cos\theta=\frac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|}=\frac{a_1b_1+a_2b_2+a_3b_3}{|\vec{A}|.|\vec{B}|}$

$=\frac{2\times4+4\times2-4\times4}{|\vec{A}|.|\vec{B}|}=0$

$\therefore \theta = \cos^{-1}(0°) \Rightarrow \theta = 90°$

14. (b) $|\vec{A}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4+9+1} = \sqrt{14}$

$|\vec{B}| = \sqrt{(-1)^2 + 3^2 + 4^2} = \sqrt{1+9+16} = \sqrt{26}$

$\vec{A}.\vec{B} = 2(-1) + 3\times 3 + (-1)(4) = 3$

The projection of $\vec{A}$ on $\vec{B}$ $= \dfrac{\vec{A}.\vec{B}}{|\vec{B}|} = \dfrac{3}{\sqrt{26}}$

15. (c) $\hat{n} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \dfrac{8\hat{i} - 8\hat{j} - 8\hat{k}}{8\sqrt{3}} = \dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

There are two unit vectors perpendicular to both $\vec{A}$ and $\vec{B}$ they are $\hat{n} = \pm\dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

16. (a) $W = \overline{F}(\overline{r}_2 - \overline{r}_1)$

$= (3\hat{i} - 2\hat{j} + \hat{k})(6\hat{i} + 8\hat{j} + 7\hat{k}) = 18 - 16 + 7 = 9\,J$

17. (d) $AB\cos\theta = AB\sin\theta \Rightarrow \tan\theta = 1 \;\therefore\; \theta = 45°$

$\therefore |\overline{C}| = \sqrt{A^2 + B^2 + 2AB\cos 45°} = \sqrt{A^2 + B^2 + \sqrt{2}AB}$

18. (c) Vector perpendicular to A and B, $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

$\therefore$ Unit vector perpendicular to A and B

$\hat{n} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}|\times|\vec{B}|\sin\theta}$

19. (c) P and Q will be parallel if $\dfrac{2}{1} = \dfrac{b}{1} = \dfrac{2}{1} \;\therefore b = 2$

20. (b) $|\vec{A}| = 5$, $|\vec{B}| = 10 \Rightarrow \dfrac{A}{B} = \dfrac{1}{2}$

21. (d) $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{B} = \hat{i} + \hat{j} + \hat{k}$

Area of the triangle $= \dfrac{1}{2}\left(\vec{A}\times\vec{B}\right)$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2}\left|2\hat{i} - 3\hat{j} + \hat{k}\right| = \frac{1}{2}\sqrt{4+9+1}$$

$= \dfrac{\sqrt{14}}{2}\,sq.unit$

22. (a) The relative velocity $v_{rel.} = 60 - (-30) = 90\,km/hr$.

Distance between the train $s_{rel.} = 90\,km$,

$\therefore$ Time when they collide $= \dfrac{s_{rel.}}{v_{rel.}} = \dfrac{90}{90} = 1\,hr$.

23. (b) If the breadth of the lake is l and velocity of boat is v_b. Time in going and coming back on a quite day

$t_Q = \dfrac{l}{v_b} + \dfrac{l}{v_b} = \dfrac{2l}{v_b}$ .....(i)

Now if v_a is the velocity of air- current then time taken in going across the lake,

$t_1 = \dfrac{l}{v_b + v_a}$ [As current helps the motion]

and time taken in coming back $t_2 = \dfrac{l}{v_b - v_a}$

[As current opposes the motion]

So $t_R = t_1 + t_2 = \dfrac{2l}{v_b[1 - (v_a/v_b)^2]}$ .....(ii)

From equation (i) and (ii)

$\dfrac{t_R}{t_Q} = \dfrac{1}{[1 - (v_a/v_b)^2]} > 1$ [as $1 - \dfrac{v_a^2}{v_b^2} < 1$] *i.e.* $t_R > t_Q$

i.e. time taken to complete the journey on quite day is lesser than that on rough day.

24. (a) $v_T = \sqrt{v_{TC}^2 + v_C^2} = \sqrt{(25\sqrt{3})^2 + (25)^2}$

$= \sqrt{1875 + 625} = \sqrt{2500} = 25\,km/hr$

25. (b)

26. (d) Let the velocity of the scooterist $= v$

Relative velocity of scooterist with respect to bus $= (v - 10)$

$\Rightarrow S = (v - 10)\times 100 \Rightarrow 1000 = (v - 10)\times 100$

$\therefore v = 10 + 10 = 20\,m/s$

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